Math·U·See

Instructor Quick Check: Question Block 13

How to Administer the Quick Check:

- The Quick Check consists of two parts: an Instructor portion which includes solutions and a Student portion with problems for each concept.
- **Your student need only complete the Quick Check problems for the concepts for which you responded Unsure.**
- Have your student complete the Quick Check items independently. You may attempt to clarify the wording of a question, but you should not provide hints about how to solve a problem.
- Return to the Question Block when you have checked your student's work.
- *You should now be able to answer **Yes** or **No** for each question.*
- Click **Next** to go to the next screen.

13.1 Can my student differentiate equations?

13.1a **Find $\frac{dw}{dx}$ given $w = \frac{4}{\sqrt{x}}$.**

$\frac{dw}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$ **or** $-\frac{1}{2\sqrt{x^3}}$

To find the derivative use the power rule after changing $\sqrt{x}$ to $x^{-\frac{1}{2}}$

$\frac{d}{dx}4x^{-\frac{1}{2}}$

$4\frac{d}{dx}x^{-\frac{1}{2}}$

$4(-\frac{1}{2})(x^{-\frac{1}{2}-1})$

$-2x^{-\frac{3}{2}}$

13.1b **Find f'' given $f(x) = \sin(x) + 3x^2$**

$-\sin x + 6$

$f'(x) = \cos x + 6x$

$f'' = -\sin x + 6$

13.2 Can my student apply the Chain Rule?

13.2a **Find $\frac{dy}{dx}$ given $y = 1 - u^2$ and $u = 2x$**

$$\frac{dy}{dx} = -8x$$

To find the derivative, remember $\frac{dy}{du}\left(\frac{du}{dx}\right) = \frac{dy}{dx}$

First find $\frac{dy}{du} = -2u$

$\frac{du}{dx} = 2$

Multiply: $-4u = -4(2x) = -8x$

$\frac{dy}{dx} = -8x$

13.2b **Find $\frac{dy}{dx}$ given $y = 4u^2$, $u = 3 - 2t$, $t = 2x$**

$$\frac{dy}{dx} = -96 + 128x$$

$\frac{dy}{du}\frac{du}{dt}\frac{dt}{dx} = \frac{dy}{dx}$

$\frac{dy}{du} = 8u$

$\frac{du}{dt} = -2$

$\frac{dt}{dx} = 2$

$(8u)(-2)(2) = -32u = -32(3 - 2t) = -96 + 64t = -96 + 64(2x) = -96 + 128x$

13.3 Can my student integrate equations?

13.3a **Integrate $\int 3x^2(2 - x^3)dx$**

$\mathbf{2x^3 - \frac{1}{2}x6}$

Let u = $x^3 \frac{du}{dx} = 3x^2$

$du = 3x^2\ dx$

Substitute in the equation $\int (2 - u)\ du = 2u - \frac{1}{2}u2$

Plug in x^3 for u,

$2x^3 - \frac{1}{2}x^6$

13.3b **Integrate $\int_{-1}^{2} - y^2 + y + 2dy$**

$\mathbf{4\frac{1}{2}}$ **or 4.5**

$-\frac{1}{3}y^3 + \frac{1}{2}y^2 + 2y$ from -1 to 2

$-\frac{1}{3}(2)^3 + \frac{1}{2}(2)^2 + 2(2) - [-\frac{1}{3}(-1)^3 + \frac{1}{2}(-1)^2 + 2(-1)]$

$-\frac{8}{3} + 2 + 4 - [\frac{1}{3} + \frac{1}{2} - 2]$

$6 - \frac{8}{3} - \frac{1}{3} - \frac{1}{2} + 2$

$= \frac{9}{2}$ or $4\frac{1}{2}$ or 4.5

13.4 Can my student apply the Mean Value Theorem and L'Hôpital's Rule?

13.4a **Evaluate** $\lim\limits_{x \to 0} \dfrac{x^2 + \cos(x)}{2x}$

0

Take the derivative of the numerator f (x) and the denominator g(x).

$f'(x) = 2x - \sin(x)$

$g'(x) = 2$

$\lim\limits_{x \to 0} \dfrac{2x - \sin(x)}{2} = \dfrac{0 + 0}{2} = 0$

13.4b **Find a value of c which satisfies the MVT for $f(x) = \sqrt{x-1}$ on [1, 5]?**

$\mathbf{x = 2}$

For the point $x = 1$, $f(x) = 0$

For the point $x = 5$, $f(x) = 2$

The slope is $\dfrac{(2-0)}{(5-1)} = \dfrac{2}{4} = \dfrac{1}{2}$

Find the derivative of $f(x) = \sqrt{x - 1 = (x-1)^{\frac{1}{2}}}$

$f'(x) = \frac{1}{2}(x-1)^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x-1}}$

$\dfrac{1}{2\sqrt{x-1}} = \dfrac{1}{2}$

$\sqrt{x-1} = 1$

$x - 1 = 1$

$x = 2$, so $c = 2$

13.5 Can my student solve differential equations?

13.5a **Solve $\frac{dy}{dx} = e^y \sin(x)$ for x**

$\mathbf{y = -\ln[\cos(x) + C]}$

$dy = e^y \sin(x) dx$

$\frac{dy}{e^y} = \sin(x) dx$

$\int \frac{\text{dy}}{e^y} = \int \sin(x) dx$

$e^{-y} dy = \sin(x) dx$

Let $u = -y$

$du = -1dy$

$-1du = dy$

$\int -1e^u du = \int \sin(x) dx$

$-1e^u = -\cos(x) + C$

$e^u = \cos(x) + C$

$\ln(e^u) = \ln[\cos(x) + C]$

$u = \ln[\cos(x) + C]$

$-y = \ln[\cos(x) + C]$

$y = -\ln[\cos(x) + C]$

13.5 Can my student solve differential equations?

13.5b **Solve $\frac{dy}{dx}\cos(2x) = \sin(2x)$ when $y(0) = 1$**

$\mathbf{c = 1}$

$dy = \frac{\sin(2x)}{\cos(2x)}dx$

$\int dy = \int \frac{\sin(2x)}{\cos(2x)}dx$

Let $u = \cos(2x)$

$du = 2\sin(2x)dx$

$-\frac{1}{2}du = \sin(2x)dx$

$\int dy = \int -\frac{1}{2} \int \frac{1}{u}du$

$y = -\frac{1}{2}\ln(u) + C$

$y = -\frac{1}{2}\ln[\cos(2x)] + C$

When $y(0) = 1$

$1 = -\frac{1}{2}\ln\{\cos[2(0)]\} + C$

$1 = -\frac{1}{2}\ln[\cos(0)] + C$

$1 = -\frac{1}{2}\ln[\cos(0)] + C$

$1 = -\frac{1}{2}\ln(1) + C$

$1 = -\frac{1}{2}(0) + C$

$1 = C$

How to complete the Quick Check:

- You only need to complete the problems your parent or instructor assigns.

13.1

13.1a **Find $\frac{dw}{dx}$ given $w = \frac{4}{\sqrt{x}}$.**

13.1b **Find f'' given $f(x) = \sin(x) + 3x^2$**

13.2

13.2a **Find $\frac{dy}{dx}$ given $y = 1 - u^2$ and $u = 2x$**

13.2b **Find $\frac{dy}{dx}$ given $y = 4u^2$, $u = 3 - 2t$, $t = 2x$**

13.3

13.3a Integrate $\int 3x^2(2 - x^3)dx$

13.3b Integrate $\int_{-1}^{2} - y^2 + y + 2dy$

13.4

13.4a **Evaluate** $\lim_{x \to 0} \frac{x^2 + \cos(x)}{2x}$

13.4b **Find a value of** c **which satisfies the MVT for** $f(x) = \sqrt{x-1}$ **on** $[1, 5]$**?**

13.5

13.5a Solve $\frac{dy}{dx} = e^y \sin(x)$ for x

13.5

13.5b **Solve $\frac{dy}{dx}\cos(2x) = \sin(2x)$ when $y(0) = 1$**