Instructor Quick Check: Question Block 12

How to Administer the Quick Check:

- The Quick Check consists of two parts: an Instructor portion which includes solutions and a Student portion with problems for each concept.
- **Your student need only complete the Quick Check problems for the concepts for which you responded Unsure.**
- Have your student complete the Quick Check items independently. You may attempt to clarify the wording of a question, but you should not provide hints about how to solve a problem.
- Return to the Question Block when you have checked your student's work.
- *You should now be able to answer **Yes** or **No** for each question.*
- Click **Next** to go to the next screen.

12.1 Can my student convert easily between radian and degree measure?

12.1a **Without using a calculator, determine how many degrees are equivalent to 1.5π radians.**

270°

1.5π radians is $\frac{3}{4}$ of a full turn ($1.5\pi = \frac{3\pi}{2}$ and $\frac{3\pi}{2} \div \frac{2\pi}{1} = \frac{3}{4}$) which is equivalent to a 270° angle ($\frac{3}{4} = \frac{270°}{360°}$).

Alternatively, you could convert using a factor like $\frac{180°}{\pi}$, thus $\frac{3\pi}{2} \times \frac{180°}{\pi} = 270°$

12.1b **Without a calculator, determine how many radians are equivalent to 280°.**

$\frac{14\pi}{9}$ radians

A 280° angle is equivalent to $\frac{280°}{360°} = \frac{7}{9}$ of a full turn and a full turn is 2π radians, so a 280° angle is equivalent to $2\pi \times \frac{7}{9} = \frac{14\pi}{9}$ radians.

Alternatively, using a conversion factor, $\frac{280°}{1} \times \frac{\pi}{180°} = \frac{14\pi}{9}$

This answer could also be written in other ways, such as $1\,\frac{5\pi}{9}$ radians or 1.5π radians or even (if they understand that $2\pi = \tau$) $\frac{7\tau}{9}$ radians.

Note: A rounded decimal answer such as 4.88692, while correct, could indicate that the student used a calculator or an online tool to quickly convert between degrees and radians. While it is fine to use a calculator to save time when working on more complex problems, you should ask the student to solve this particular problem without using a calculator or online tool in order to make sure they really understand the relationship between degrees and radians.

12.2 Can my student convert polar equations to rectangular equations and vice versa?

12.2a **Express $x^2 + y^2 = 25$ with polar coordinates using the variables r and/or θ.**

$\boldsymbol{r = 5}$

One way to solve this would be to recognize $x^2 + y^2 = 25$ (either by remembering the standard form of a circle or by sketching the graph) as the equation of a circle centered on the origin with a radius of 5. Such a circle can be expressed in polar coordinates as $r = 5$.

Another way to solve it would be to remember that $r\cos\theta = x$ and $r\sin\theta = y$, which would mean:

$x^2 + y^2 = 25$

$(r\cos\theta)^2 + (r\sin\theta)^2 = 25$

$r^2\cos^2\theta + r^2\sin^2\theta = 25$

$r^2(\cos^2\theta + \sin^2\theta) = 25$

If they remember that $\cos^2\theta + \sin^2\theta = 1$, they can substitute that giving: $r^2 = 25$

This is a valid answer, however, they could also take the square root of both sides giving $\pm r = 5$; which, if they recognize that there is no angle in this equation and r is a distance (in other words $+r = -r$ when there is no θ), can be simplified to $r = 5$.

12.2b **Express $r\sin\theta - 2r\cos\theta = 0$ with rectangular coordinates using the variables x and/or y.**

$\boldsymbol{y = 2x}$

One way to solve this would be to remember that $r\cos\theta = x$ and $r\sin\theta = y$, substituting these variables could give an algebraic solution like this:

$r\sin\theta - 2r\cos\theta = 0$

$y - 2x = 0$

$y = 2x$

Another way to solve this would be to sketch $r\sin\theta - 2r\cos\theta = 0$ and recognize it as a line with a slope of 2 and a y-intercept of 0, therefore $y = 2x$.

12.3a

Can my student identify parts of a quadratic function responsible for translation, direction, and amplitude?

12.3a **Without using a calculator or online tool, identify the parabola which has the highest vertex.**

- [ ] $f(x) = -2x^2 + 3$
- [x] $f(x) = (x-1)^2 + 5$
- [ ] $f(x) = x^2 - 8x + 20$

If a student recalls that the vertex of a parabola expressed as $f(x) = a(x-h)^2 + k$ is (h, k), they could rewrite each of the equations above in that form.

$f(x) = -2x^2 + 3$	$f(x) = (x-1)^2 + 5$	$f(x) = x^2 - 8x + 20$
$f(x) = (-2x - 0)^2 + 3$	vertex = (1, 5)	$f(x) = (x-4)^2 + 4$
vertex = (0, 3)		vertex = (4, 4)

Of these vertices, the highest is (1, 5) which was the vertex of $f(x) = (x-1)^2 + 5$.

For the third equation (which isn't in vertex form), they could use the formula for the axis of symmetry $x = \frac{b}{2a}$ with the standard form of the parabola equation $f(x) = ax^2 + bx + c$. Using the values from $f(x) = x^2 - 8x + 20$, this would tell that the x- value of the vertex for that equation is $x = \frac{8}{2} = 4$, and using $x = 4$ in the original function gives us $f(x) = (4)^2 - 8(4) + 20 = 4$, thus the vertex of $f(x) = x^2 - 8x + 20$ is (4, 4).

If the student cannot remember either the formula for the axis or the vertex form of the equation, sketching a graph may help them recall that the portion of the equation responsible for the height of the vertex is the k in $f(x) = a(x-h)^2 + k$.

12.3b **Without using a calculator or online tool, identify the parabola which is the widest (greatest amplitude).**

- [x] $f(x) = -2x^2 + 3$
- [ ] $f(x) = (x-1)^2 + 5$
- [ ] $f(x) = x^2 - 8x + 20$

Students may recall that the amplitude of a parabola in the form $f(x) = a(x-h)^2 + k$ is a, thus the amplitudes for the parabolas above are (in order), −2, 1, and 1.

Alternatively, they could quickly sketch these parabolas and note that the first parabola is the widest, or they could compare simpler examples like $f(x) = \frac{1}{2}x^2$ and $f(x) = 2x^2$ with the basic form $f(x) = x^2$.

12.4 Can my student identify parts of a trigonometric function responsible for amplitude, translation, and period?

12.4a **Without using a calculator or online tool, identify the sine function which has the longest period.**

- [x] **$y = -2\sin(x) + 2$**
- [ ] **$y = \sin(4x) - 3$**
- [ ] **$y = 2\sin(3x) + 1$**

The best way to approach this is to look at the generic formula and which element has which effect on the final graph.

If we express the generic formula as $y = A\text{fun}(P(x - S)) + T$, the amplitude of the function is $|A|$, the phase (horizontal) shift is S, and the translation (vertical shift) is T. P represents the frequency of the function (how many times it repeats in the space of 2π), so the period (the length in which it repeats) is $2\pi/P$.

We can then quickly calculate the frequency and period for each of the sine functions listed.

$y = -2\sin(x) + 2$; frequency $= 1$, period $= \frac{2\pi}{1} = 2\pi$.

$y = \sin(4x) - 3$; frequency $= 4$, period $= \frac{2\pi}{4} = \frac{\pi}{2}$ or 0.5π

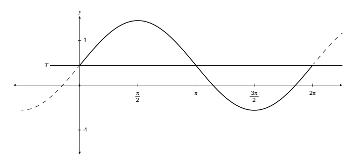
$y = 2\sin(3x) + 1$; frequency $= 3$, period $= \frac{2\pi}{3}$ or $0.\overline{6}\pi$

$2\pi > \frac{2\pi}{3} > \frac{\pi}{2}$ so the function with the longest period is

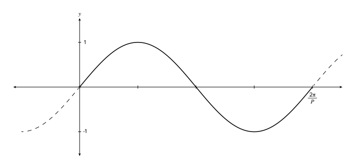
$y = -2\sin(x) + 2$

A student who recognizes that 2π is a constant in all these formulas may just compare $\frac{1}{1}$ to $\frac{1}{4}$ to $\frac{1}{3}$ to come to the same conclusion. Another (more time-consuming) option for those who don't remember the generic formula or just need to check their memory would be to quickly sketch one or more of the functions and measure the period from the graph.

Note: A common error is confusing the period with the frequency. A student who thought the second function had the largest period and thought that the period was 4 may just need a quick review of trigonometric functions before moving on to Calculus.

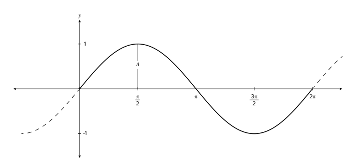


Translation $= T$
Moves the graph up or down vertically. Follows the sign.

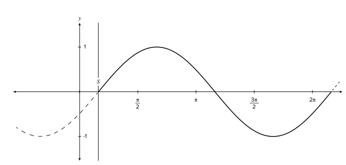


Frequency $= P$
How many times the cycle repeats in a space of 2π.

Period $= 2\pi/P$
The distance it takes to complete one complete cycle.



Amplitude $= |A|$
Maximum distance from the horizontal baseline (T).



Shift $= S$
Moves the graph left or right horizontally. Placing S in parentheses helps show direction.

12.4 Can my student identify parts of a trigonometric function responsible for amplitude, translation, and period?

12.4b **Without using a calculator or online tool, identify the sine function which has the smallest amplitude.**

- [ ] **$y = -2\sin(x) + 2$**
- [ ] **$y = \sin(4x) - 3$**
- [ ] **$y = 2\sin(3x) + 1$**

Again, the easiest approach will be to look at each in terms of the generic formula $y = A\text{fun}(P(x - S)) + T$.

$y = -2\sin(x) + 2$; amplitude = $|-2| = 2$

$y = \sin(4x) - 3$; amplitude = $|1| = 1$

$y = 2\sin(3x) + 1$; amplitude = $|2| = 2$

Although −2 is the number with the lowest value, the smallest distance (which is an absolute value) created by these values is 1, so the function with the smallest amplitude will be $y = \sin(4x) - 3$

Note: A common error is not thinking of amplitude or other measures of size in terms of absolute value. A student who thought the first function had the smallest amplitude and thought that the amplitude was −2 may just need a quick review of trigonometric functions and/or absolute value before moving on to Calculus.

12.5 Can my student work with expressions which contain trigonometric functions, logarithms, radical expressions, irrational numbers, and/or complex numbers?

12.5a **Solve for x: $\text{Log}_2 8 = \sqrt{x}$**

$x = 9$

The student should be able to recall that $\text{Log}_2 8$ refers to the power to which 2 must be raised to equal 8, therefore $\text{Log}_2 8 = 3$ because $2^3 = 8$. From there, solving for x is a simple matter of squaring both sides of the equation.

$3 = \sqrt{x}$

$(3)^2 = (\sqrt{x})^2$

$9 = x$

12.5b **Which of the following is equal to $\frac{b^2 b^{-3} c^4}{c^{-2} b}$?**

- [x] $\frac{c^6}{b^2}$
- [ ] $\frac{b^2}{c^6}$
- [ ] $c^4 b^{-2}$
- [ ] $c^{-4} b^2$

One approach to solving this would be to rewrite each of the expressions on a single line using negative exponents where necessary and putting the variables in alphabetical order.

$\frac{c^6}{b^2} = b^{-2}c^6$

$\frac{b^2}{c^6} = b^2c^{-6}$

$c^4b^{-2} = b^{-2}c^4$

$c^{-4}b^2 = b^2c^{-4}$

$\frac{b^2 b^{-3} c^4}{c^{-2} b} = b^2 b^{-3} b^{-1} c^4 c^2$

Recall that when multiplying factors with the same base we can add the exponents.

$2 + (-3) + (-1) = -2$ and $4 + 2 = 6$, so $b^2 b^{-3} b^{-1} c^4 c^2 = b^{-2} c^6$, which, as we see above, is equal to $\frac{c^6}{b^2}$.

Another approach is to express everything as a rational expression (a.k.a. fraction) and simplify each expression by dividing the numerator and denominator by common factors (a.k.a. cancelling).

$\frac{c^6}{b^2} = \frac{c^6}{b^2}$

$\frac{b^2}{c^6} = \frac{b^2}{c^6}$

$c^4b^{-2} = \frac{c^4}{b^2}$

$c^{-4}b^2 = \frac{b^2}{c^4}$

$\frac{b^2 b^{-3} c^4}{c^{-2} b} = \frac{c^6}{b^2}$

How to complete the Quick Check:

- You only need to complete the problems your parent or instructor assigns.

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- ☐ $c^{-4}b^2$