Instructor Quick Check: Question Block 11

How to Administer the Quick Check:

- The Quick Check consists of two parts: an Instructor portion which includes solutions and a Student portion with problems for each concept.
- **Your student need only complete the Quick Check problems for the concepts for which you responded Unsure.**
- Have your student complete the Quick Check items independently. You may attempt to clarify the wording of a question, but you should not provide hints about how to solve a problem.
- Return to the Question Block when you have checked your student's work.
- *You should now be able to answer **Yes** or **No** for each question.*
- Click **Next** to go to the next screen.

11.1 Can my student work with multiple-degree rational expressions?

11.1a **Which of the following is equivalent to $\frac{(5x^2 - 5a^2)}{5a}$?**

- $5x^4$
- $\frac{(5ax^2)}{5a}$
- $\boxed{\frac{(x^2 - a^2)}{a}}$
- $x^2 - a$

Rewriting by removing common factors

$$\frac{(5x^2 - 5a^2)}{5a} = \frac{(5(x^2 - a^2))}{(5(a))} = \frac{(x^2 - a^2)}{a}$$

11.1b **Which of the following is equivalent to $\frac{3i}{(2 + 8i)}$?**

- $\boxed{\frac{(3i + 12)}{34}}$
- -0.3
- $\frac{-2i}{10}$
- $0.4i$

Rewriting using the conjugate of the expression in the denominator

$$\frac{3i}{(2 + 8i)} = \frac{(3i(2 - 8i))}{((2 + 8i)(2 - 8i))} = \frac{(6i - 24i)}{(4 - 64i)} = \frac{(6i - (-24))}{(4 - (-64))} = \frac{(6i + 24)}{(4 + 64)} = \frac{(6i + 24)}{68} = \frac{(2(3i + 12))}{(2(34))} = \frac{(3i + 12)}{34}$$

11.2 Can my student solve quadratic equations with irrational or complex roots by completing the square?

11.2a **Without using a calculator, online tool, or the quadratic formula, solve $x^2 - 2x - 11 = 0$ by completing the square.**

$\mathbf{x = 1 \pm 2\sqrt{3}}$

$x^2 - 2x - 11 = 0$

$x^2 - 2x + \_\_\_ = 11 + \_\_\_$ (Adding the same value to both sides, to keep the equation balanced.)

$x^2 - 2x + \ 1 \ = 11 + \ 1$ (We found 1, by taking −2 from −2x, dividing by 2, and then squaring.)

$(x - 1)^2 = 12$

$x - 1 = \pm\sqrt{12}$

$x = 1 \pm \sqrt{12}$

$x = 1 \pm 2\sqrt{3}$

11.2b **Without using a calculator, online tool, or the quadratic formula, solve $x^2 - 10x + 30 = 0$ by completing the square.**

$\mathbf{x = 5 \pm i\sqrt{5}}$

$x^2 - 10x + 30 = 0$

$x^2 - 10x + \_\_\_ = -30 + \_\_\_$

$x^2 - 10x + 25 = -30 + 25$ $(\frac{-10}{2} = -5, \ (-5)^2 = 25)$

$(x - 5)^2 = -5$

$x - 5 = \pm\sqrt{-5}$

$x - 5 = \pm i\sqrt{5}$ (i can be written before or after $\sqrt{5}$, but not under the line with the radical sign.)

$x = 5 \pm i\sqrt{5}$

Note: Some other math curricula may have only taught a student to use the quadratic formula and will not have built a solid understanding of how and why the formula works the way that it does or have taught the student how to use alternative methods of solving quadratic equations such as completing the square. We believe it is important to have a solid understanding of how quadratic equations work which goes beyond simply memorizing a formula. If a student is only able to solve 11.2a and 11.2b using the quadratic formula but has all the other skills in Question Block #11 they may be able to move forward after a quick review of relevant concepts.

11.3b

Can my student use discriminants and an understanding of the quadratic formula to predict the nature of the solution to a quadratic equation?

11.3a

Without fully solving the equation, predict what type of solutions you would find to the following equation (i.e. will they be real or complex, rational or irrational, and equal or unequal):
$\mathbf{4x^2 - 20x + 25 = 0}$

The solutions will be real, rational, and equal.

The quadratic formula states that $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$ for all equations in the form $ax^2 + bx + c = 0$.

The discriminant (the part which determines what type of solutions the equation will have) is $b^2 - 4ac$. If the discriminant is positive, then all values of x will be real, if the discriminant is a perfect square, then all values of x will be rational, and if the discriminant is zero, the solutions will be equal (i.e. there will only be one solution). Evaluating just the discriminant can therefore tell us what the solutions will be like without needing to solve the rest of the equation.

For $4x^2 - 20x + 25 = 0$ the discriminant is $(-20)^2 - 4(4)(25)$.
This = 0, so the solutions will be real, rational, and equal.

11.3b

Without fully solving the equation, predict what type of solutions you would find to the following equation (i.e. will they be real or complex, rational or irrational, and equal or unequal):
$\mathbf{x^2 - 2x - 3 = 0}$

The solutions will be complex (nonreal), irrational, and unequal.

For $x^2 - 2x - 3 = 0$ the discriminant is $12 - 4(-2)(-3)$.

This = −23, so the solutions will be complex (nonreal), irrational, and unequal.

11.4 Can my student understand and apply the binomial theorem?

11.4a **Without fully multiplying it out, how many terms will there be in $(x - 2)^5$?**

6 terms

One more than the value of the exponent is the number of terms in a binomial raised to any power. Counting from one to the value of the exponent, and then adding one more term for zero is how we know the number of terms in any binomial product.

In the case of $(x - 2)^5$ there will be 6 terms $(5 + 1)$.

11.4b **Without fully multiplying it out, what will the third term be in $(x - 2)^5$?**

$40x^3$

The binomial theorem describes the pattern which emerges when a binomial is raised to a given power.

$$(a + b)^n = a^n b^0 + \frac{(n-1)}{1}a^{n-1}b^1 + \frac{(n(n-1))}{(1(2))}a^{n-2}b^2 + \frac{(n(n-1)(n-2))}{(1(2)(3))}a^{n-3}b^3 \; [\ldots] \; + a^{n-n}b^n$$

The third term of $(a + b)^n$ is $\frac{(n(n-1))}{(1(2))}a^{n-2}b^2$ which in the case of $(x - 2)^5$ is $\frac{(5(5-1))}{(1(2))}(x)^{5-2}(-2)^2$.

$$\frac{(5(5-1))}{(1(2))}(x)^{5-2}(-2)^2 = \frac{(5(4))}{(1(2))}(x)^3(-2)^2 = \frac{20}{2}x^3(-2)(-2) = 10x^3(4) = 40x^3$$

Note: There may be other ways to solve this problem, e.g. by using Pascal's triangle or another formula to predict the binomial coefficient.

How to complete the Quick Check:

- You only need to complete the problems your parent or instructor assigns.

11.1

11.1a **Which of the following is equivalent to $\frac{(5x^2 - 5a^2)}{5a}$?**

- [ ] $5x^4$
- [ ] $\frac{(5ax^2)}{5a}$
- [ ] $\frac{(x^2 - a^2)}{a}$
- [ ] $x^2 - a$

11.1b **Which of the following is equivalent to $\frac{3i}{(2+8i)}$?**

- [ ] $\frac{(3i+12)}{34}$
- [ ] -0.3
- [ ] $\frac{-2i}{10}$
- [ ] $0.4i$

11.2

11.2a **Without using a calculator, online tool, or the quadratic formula, solve $x^2 - 2x - 11 = 0$ by completing the square.**

11.2b **Without using a calculator, online tool, or the quadratic formula, solve $x^2 - 10x + 30 = 0$ by completing the square.**

11.3

11.3a **Without fully solving the equation, predict what type of solutions you would find to the following equation (i.e. will they be real or complex, rational or irrational, and equal or unequal):**
$4x^2 - 20x + 25 = 0$

11.3b **Without fully solving the equation, predict what type of solutions you would find to the following equation (i.e. will they be real or complex, rational or irrational, and equal or unequal):**
$x^2 - 2x - 3 = 0$

11.4

11.4a Without fully multiplying it out, how many terms will there be in $(x - 2)^5$?

11.4b Without fully multiplying it out, what will the third term be in $(x - 2)^5$?